UNIVERSITÉ PARIS 1
PANTHÉON SORBONNE

Financial Products: exercises

Corrected version

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Contents

1 Exercises **5**
1.1 Forward contracts 5
1.2 Options 8

2 Problems **17**
2.1 The Merton model of the firm 17
2.2 Structured Products 20
2.3 Hedging a Corporate FX Exposure 21
2.4 Multi-period / multi-currency financing 24
2.5 Benefiting from the Abenomics 27

Foreword

The following exercises have been selected from various online sources to help you practice the notions introduced in the lecture "Financial Products and Introduction to Pricing". They are also included, with the same solutions, in the lecture notes.

Chapter 1

Exercises

Unless otherwise specified, interest rates are continuously compounded and expressed per annum: the discount factor for a rate r and a maturity T is e^{-rT}.

1.1 Forward contracts

Exercise 1

A share currently trades at 80 USD. Interest rates are equal to 5%, regardless of the maturity. A dividend will be paid in 6 months. Determine the forward price for delivery in 9 months when:

(a) the dividend equals 0 USD;

(b) the dividend equals 3 USD;

(c) the dividend equals 4% of the value of the stock at that time.

Solution 1

(a) 83.06 USD

(b) 80.02 USD

(c) 79.73 USD

Exercise 2

The current forward price of a share to be delivered in six months equals 58 USD. Interest rates are equal to 10% per annum, regardless of the maturity. Determine the spot price if:

(a) no intermediate dividend is paid;

(b) a dividend of 2 USD is paid in 3 months;

(c) a dividend equal to 8% of the value of the stock at that time is paid in 3 months;

(d) a dividend equal to 8% of the value of the stock at that time is paid in 6 months.

Solution 2

(a) 55.17 USD

(b) 57.12 USD

(c) 59.97 USD

(d) 59.97 USD

Exercise 3

The exchange rate of US dollars is today 8.50 SEK per dollar. The forward price of a dollar to be delivered in six months is 8.40 SEK. If the Swedish six month interest rate is 4%, determine the US six month interest rate.

Solution 3

6.37%.

Exercise 4

We are August 1^{st}. The forward price of a US dollar to be delivered on December 31^{st} is 0.94630 Euros. The forward price of a US dollar to be delivered at the end of June next year is 0.95152 Euros. Assuming a flat term structure in both countries and that the Euro interest rate equals 4%, what is the level of US rates?

Solution 4

2.90%.

Exercise 5

The S&P index spot price is 1100 and the 9-month zero-coupon bond price, in % terms, is $ZC = 97.00$. You observe a 9-month forward price of 1109. The dividend is supposed to be paid right before the forward contract maturity.

(a) What dividend value is implied by this forward price?

(b) Suppose you believe the dividend that will be paid will be equal to 15. What strategy would you undertake?

Solution 5

(a) Let D be the value of the dividend. By absence of arbitrage, $1109 = \frac{1100}{0.97} - D$, hence $D = 1134 - 1109 = 25$.

(b) If the dividend paid is 15 instead of 25, the forward price should be $1134 - 15 = 1119$: the forward is too cheap. One should go long the forward, sell the index short and lend the proceeds, which yields a profit of 10 at maturity.

Exercise 6

Boeing just signed a contract to sell a 777 aircraft to Air France. Air France will be billed \$300 million, payable in one year. Air France has the major part of its cash in euros. The current spot exchange rate is 1.090\$/€ and the one-year forward rate is 1.112\$/€. The annual interest rate available to Air France is 5.0% in the U.S. and 3.0% in Europe. Air France is concerned with the volatile exchange rate between the dollar and the euro and would like to hedge its currency exposure.

(a) Describe qualitatively the risk which Air France is taking.

(b) Air France is considering a first hedging alternative, which would consist of buying forward the amount of USD to be paid, thanks to a currency forward contract. What would be the final euro value of such an operation?

(c) The second hedging alternative would consist of borrowing euros, changing them spot into USD, and lending these USD so that the amount received in 1 year is exactly \$300M. What would be the final euro value of such an operation?

Solution 6

(a) Since Air France will have to pay USD in one year, its risk is that the USD appreciates against the EUR.

(b) Buying the USD forward costs, in euros, $V_1 = \$300,000,000/1.112 = €269,784,173$.

(c)
- Lending USD: $300,000,000 \times e^{-0.05} = \$285,368,827$;
- Borrowing the corresponding euros and changing them spot: $\$285,368,827/1.09 = €261,806,264$;
- Repaying the euro loan in one year: $€261,806,264 \times e^{0.03} = €269,779,451$.

Both alternatives cost the same, up to the rounding of the forward rate: this is the cash and carry argument of the forward exchange rate.

1.2 Options

Exercise 7

(Use a B&S calculator available on the Web.) Determine the price of a European futures call option on a barrel of crude oil to be delivered in 4 months, which is also the maturity of the option. The futures price today is $F_0 = \$25.00$, the strike price is \$23.00 and the volatility of the futures price is estimated to be 25% per year. The risk-free interest rate is 9% per year.

Solution 7

The appropriate formula is the Black formula for futures. It can be obtained from a Black and Scholes calculator with the following parameters:

- spot = forward = 25;
- strike, maturity and rate at their level;
- dividend yield = rate, to ensure that the forward price is equal to the spot price.

The price is 2.53.

Exercise 8

(Use a B&S calculator available on the Web.)

(a) Calculate the price of a European call option on a share which does not pay dividends before the maturity of the option. The spot share price is 45 SEK, the option matures in 6 months, the volatility is 25% per annum, the strike price is 43 SEK and the risk-free interest rate is 5% per annum.

(b) Same question, assuming now that the share pays a dividend of 1 SEK in 3 months, all other assumptions being unchanged.

Solution 8

(a) 4.832

(b) 4.176

Exercise 9

Denote X_t the USD vs EUR exchange rate at date t, i.e. the value of 1 USD in EUR: 1 USD $= X_t$ EUR. We denote $r_{€}$ and $r_{\$}$ respectively the EUR and the USD continuously compounded rates. In what follows, $F(t,T)$ denotes the forward value of X_t for date $T \geq t$. EUR is the domestic currency in which all prices are denominated.

(a) Give the definition of the forward price $F(t,T)$. At which date is it known? At which date is it paid?

(b) Show that $F(t,T) = X_t e^{(r_{€} - r_{\$})(T-t)}$.

We now wish to price a call with strike K and maturity T on X_T. Instead of recalculating everything, we would like to adapt the Black and Scholes formula.

(c) Use the previous formula to find the price of a call option on the exchange rate as a function of $r_{€}$, $r_{\$}$, X_0, σ and T.

(d) Compare the intrinsic value of the call and the amount of money you could get by exercising it. Discuss when it makes sense to exercise it or not.

(e) Same question for the put.

Solution 9

(a) The forward price is a quantity known today (t), paid at date T in exchange for a random quantity, with no initial exchange of money.

(b) Cash and carry arbitrage, see the section on the forward exchange rate.

(c) Apply the Black formula with $t = 0$, $r = r_{€}$ and the forward price:

$$C = e^{-r_{€}T} \left[F_T N(d_1) - K N(d_2) \right]$$

where

$$F_T = X_0 e^{(r_{€} - r_{\$})T}$$
$$d_1 = \frac{\ln(F_T/K) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

Hence, the foreign rate plays the role of a dividend yield.

(d) The value of the call is always above its intrinsic value (convexity):

$$C > \left(X_0 e^{-r_{\$}T} - K e^{-r_{€}T} \right)^+ .$$

The value obtained when exercising an American call is

$$C_{ex} = X_0 - K.$$

For options on stocks without dividends, necessarily $C > C_{ex}$, so that it is always better to sell the call than to exercise it. In the FX case, if the foreign rate $r_{\$}$ is high enough, the intrinsic value can fall below $X_0 - K$, and it might be better to exercise the call than to sell it.

Exercise 10

You are given the following:

- the current price of one share of XYZ stock is 500;
- the stock does not pay dividends;
- interest rates are null;
- a European call option on one share of XYZ stock with a strike price of K that expires in one year costs 67;
- a European put option on one share of XYZ stock with a strike price of K that expires in one year costs 19.

Find K.

Solution 10

Because interest rates are null, the price of the zero-coupon bond with notional K is K. By the call-put parity,

$$67 - 19 = 500 - K$$

hence $K = 452$.

Exercise 11

The PS index has the following characteristics:

- one share of the PS index currently sells for 1,000;
- the PS index does not pay dividends.

Sam wants to secure the ability to buy this index in one year for a price of 1,025 (even if the price eventually gets lower than 1,025). He can do so by buying or selling European put and call options with a strike price of 1,025. The annual effective risk-free interest rate is 5%. Determine which of the following gives the hedging strategy that will achieve Sam's objective, and calculate the cost today of establishing this position.

1. Buy the put and sell the call, receive 23.81
2. Buy the call and sell the put at no cost
3. Buy the call and sell the put, receive 23.81
4. Buy the call and sell the put, pay 23.81

Instead of 1,025, which strike would allow to secure the forward purchase at zero cost?

Solution 11

A long call and a short put with the same strike provide an exposure to one unit of the index, together with a borrowing of the present value of the strike. By the call-put parity, the cost is

$$Call - Put = 1000 - \frac{1025}{1.05} \simeq 23.81,$$

which corresponds to answer 4. The strike which allows to secure the purchase at zero cost is the forward price:

$$1000 - \frac{K}{1.05} = 0 \Leftrightarrow K = 1050.$$

Exercise 12

Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike price	Call price	Put price
40	11	3
50	6	8
55	3	11

Explain why this pricing table would provide some arbitrage. Propose an arbitrage strategy. (Hint: use the call-put parity.)

Solution 12

Since we are close to the maturity, we can make the approximation $Ke^{-rT} \simeq K$. We can thus complete the table and compute which stock price would allow the call-put parity to hold:

Strike price	Call price	Put price	Call - Put + Strike
40	11	3	48
50	6	8	48
55	3	11	47

Since the stock price cannot be equal to both 48 and 47, this option price table is inconsistent and can be arbitraged. An arbitrage strategy consists of buying what is cheap and selling what is expensive. By the call-put parity, a (long call, short put, long zero-coupon) position replicates the underlying.

With strike 55, this costs 47, whereas with strike 50 (or 40) it costs 48. Buying this position with strike 55 and selling it with strike 40, you receive 1 today and have no other cash flow: this is an arbitrage.

Exercise 13

Let X_t be the value of a foreign currency expressed in units of local currency. Let r_f be the (constant) interest rate of the foreign currency and r_l the local interest rate. Let $T \geq t$ be a maturity.

(a) Compute the forward value of X_t.

  (a) Does it increase or decrease with r_f? With r_l?

  (b) What is the "cost of carry" of such an "asset"?

  (c) What would be the risk neutral drift of that exchange rate?

(b) Write the equivalent of the Black and Scholes formula in such a situation (solution: `https://en.wikipedia.org/wiki/Foreign_exchange_option`).

Solution 13

The cash and carry arbitrage consists of borrowing in local currency, changing spot, carrying in foreign currency (hence receiving the foreign short rate) and changing back at maturity. As a result:

$$F(t,T) = X_t e^{(r_l - r_f)(T-t)}.$$

The forward price is the expectation of the price at T under the risk neutral probability: $E(X_T|X_t) = F(t,T)$. If X_t is modelled as $dX_t = X_t(\mu dt + \sigma dW_t)$, then

$$X_T = X_t e^{\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma(W_T - W_t)}$$

and hence

$$E(X_T|X_t) = X_t e^{\mu(T-t)}.$$

So, under the risk neutral probability, necessarily $\mu = r_l - r_f$.

Exercise 14

(a) In a world without dividends, prove the call-put parity, i.e.

$$KD(0,T) + Call(S_0, K, T, r) = S_0 + Put(S_0, K, T, r)$$

where $D(0,T) = e^{-rT}$ is the discount factor, S_0 is the initial value of the price, and $Call(S_0, K, T, r)$ (resp. $Put(S_0, K, T, r)$) is the price of the call (resp. put) of strike K and maturity T obtained with the B&S formula with interest rate r.

(b) Let F be the forward price of S: $F(t,T) = S_t e^{r(T-t)}$. Thanks to Itô's lemma, compute the drift of $F(t,T)$ under the risk neutral probability. Denote $F = F(0,T)$.

(c) Prove that, equivalently,

$$K + Call(F, K, T, 0) = F + Put(F, K, T, 0)$$

where the options are now options on the futures contract.

Solution 14

Let $\tilde{S}_T$ be the (random) price of a given stock at date T, S_0 its price today, and K a given amount of money.

Question (a). For any value of $\tilde{S}_T$,

$$\tilde{S}_T + \left(K - \tilde{S}_T\right)^+ = K + \left(\tilde{S}_T - K\right)^+ \tag{1.1}$$

since

- when $\tilde{S}_T \leq K$, both sides are equal to K;
- when $\tilde{S}_T \geq K$, both sides are equal to $\tilde{S}_T$.

If there are no intermediate flows such as coupons or dividends, by absence of arbitrage (sometimes referred to as **"the law of one price"**), the prices of both sides of equation (1.1) are equal. Since the price is linear:

$$\underbrace{Price\left(\tilde{S}_T\right)}_{1} + \underbrace{Price\left(\left(K-\tilde{S}_T\right)^+\right)}_{2} = \underbrace{Price\left(K\right)}_{3} + \underbrace{Price\left(\left(\tilde{S}_T-K\right)^+\right)}_{4}$$

This proves the statement since:

1. is the price of the underlying, S_0;
2. is the price of the put, $Put\left(S_0, K, T, r\right)$;
3. is the price of a zero-coupon bond with notional K;
4. is the price of the call, $Call\left(S_0, K, T, r\right)$.

Question (b). In the Black and Scholes model,

$$dS_t = S_t\left(rdt + \sigma dW_t\right)$$

under the risk neutral probability. By definition, the forward price is $F_t = S_t e^{r(T-t)}$, and Itô's lemma yields:

$$\begin{aligned} dF_t &= e^{r(T-t)}dS_t + S_t d\left(e^{r(T-t)}\right) \\ &= F_t\left(rdt + \sigma dW_t\right) - F_t r dt \\ &= F_t \sigma dW_t \end{aligned}$$

which shows that the forward price has a null drift (it is a martingale) under the risk neutral probability.

Question (c). This question is very similar to the first one, but instead of taking the price of both sides of equation (1.1), we take their risk neutral expectation:

$$\underbrace{\mathbb{E}\left(\tilde{S}_T\right)}_{1} + \underbrace{\mathbb{E}\left(\left(K-\tilde{S}_T\right)^+\right)}_{2} = \underbrace{\mathbb{E}\left(K\right)}_{3} + \underbrace{\mathbb{E}\left(\left(\tilde{S}_T-K\right)^+\right)}_{4}$$

Term 1 is the expected value of the spot at date T, i.e. its forward value $F = S_0 e^{rT}$. Term 2 can be related to a B&S formula with modified parameters:

$$\begin{aligned} \mathbb{E}\left(\left(K-\tilde{S}_T\right)^+\right) &= e^{rT} Put\left(S_0, K, T, r\right) \\ &= KN\left(-d_2\right) - FN\left(-d_1\right) \end{aligned} \tag{1.2}$$

with

$$\begin{aligned} d_1 &= \frac{\ln\left(S_0/K\right) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}} \\ &= \frac{\ln\left(F/K\right) + \left(\sigma^2/2\right)T}{\sigma\sqrt{T}}. \end{aligned} \tag{1.3}$$

Expressions (1.2) and (1.3) show that $\mathbb{E}\left(\left(K - \tilde{S}_T\right)^+\right)$ is the value of a put option on a futures contract in a world with no interest rates, i.e. $Put\left(F, K, T, 0\right)$. The same reasoning applies to the call. Finally, $\mathbb{E}\left(K\right) = K$, which proves the statement.

Exercise 15

Joe believes that the volatility of a stock is higher than indicated by the market prices of options on that stock. He wants to speculate on that belief by buying or selling at-the-money options. Determine which of the following strategies would achieve Joe's goal (see forwardoptionpayoffs.pdf):

1. buy a strangle;
2. buy a straddle;
3. sell a straddle;
4. buy a butterfly spread;
5. sell a butterfly spread.

Solution 15

Joe has to be long volatility, i.e. long options. The best strategy is 2 (buy an at-the-money straddle, which has the highest exposure to volatility). The second best is 1 (a strangle, with out-of-the-money options, is less exposed). Selling a straddle (3) or buying a butterfly spread (4) are short volatility positions. Selling a butterfly spread (5) is long volatility, but its gains are capped.

Chapter 2

Problems

2.1 The Merton model of the firm

Exercise 16

This model aims at providing the value of the debt of a company, taking into account its bankruptcy risk. The company is modelled as follows. Its business value at time t is denoted S_t, and its initial business value S_0. The volatility of the business is denoted σ. The company has been financed by two means:

1. some initial capital, E_0, from shareholders;
2. some initial debt in the form of a zero-coupon bond of notional D and maturity T, from creditors.

The company will not pay any intermediate dividend. The riskless interest rate is denoted r. The interest rate paid by the company equals $r+s$, where s is a spread above the riskless rate which reflects the fact that lending money to the company is risky. This means that the value of the risky zero-coupon bond today is $De^{-(r+s)T}$. In this framework, the bankruptcy event is defined as $S_T < D$.

At maturity, shareholders and creditors share the business value S_T as follows:

1. Creditors are paid first. If the company is not bankrupt, they are repaid their notional D. If the company is bankrupt, they take the whole company value.
2. Shareholders are paid second: if the company is not bankrupt, they get $S_T - D$; if it is bankrupt, they receive 0.

Let F_T be the T-forward value of the business. The debt-to-asset ratio, defined as $l = \frac{D}{F_T}$, is used as a measure of leverage. When $l = 0$, the company is fully unlevered; when it equals 1, it is infinitely leveraged.

(a) Write the final wealth of shareholders as an option payoff. Give the name of this option. Using the Black and Scholes formula, express the value of equity as a function of the interest rate r, the leverage l, the volatility σ and the maturity T.

(b) Using the fundamental law of accounting, express the value of the risky debt. Then, using the call-put parity, find an expression of the spread.

(c) Compute the value of the spread s and of the equity for the following sets of parameters, and comment:

σ	l	T	r
20%	1/2	1	0
20%	3/4	1	0
40%	1/2	1	0

(d) More generally, who has an interest in the company taking more risk ($\sigma \nearrow$)?

(e) We assume $T = 1$, $l = 3/4$, $\sigma = 30\%$ and $r = 0\%$. At date 0, the company announces that it will pay to shareholders an intermediate dividend equal to 5% of the initial value S_0. What is the new forward value of the company? What is the new leverage? Compute the new value of the share price and of the spread. Comment.

Solution 16

Question (a). Shareholders are long a call option on the firm value at date T, with strike the amount of debt to be repaid:

$$(S_T - D)^+ .$$

Hence, using the B&S formula, the value of the equity is

$$C(T, F_T) = e^{-rT} F_T \left[N(d_1) - lN(d_2) \right]$$

where

$$\begin{aligned} F_T &= S_0 e^{rT} \\ d_1 &= \frac{-\ln(l) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \\ d_2 &= \frac{-\ln(l) - \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \end{aligned}$$

Question (b). Since assets are equal to liabilities, the final payoff of creditors is

$$D^r = S_T - (S_T - D)^+ .$$

By the call-put parity, its value is

$$D^r = De^{-rT} - P(T, F_T)$$

where

$$P(T, F_T) = e^{-rT} F_T [lN(-d_2) - N(-d_1)] .$$

Hence

$$\begin{aligned} De^{-(r+s)T} &= e^{-rT} F_T [l - lN(-d_2) + N(-d_1)] \\ \iff e^{-sT} &= N(d_2) + \frac{1}{l} N(-d_1) \end{aligned}$$

$$\boxed{s = -\frac{1}{T} \ln \left(N(d_2) + \frac{1}{l} N(-d_1) \right)}$$

When leverage goes to zero:

$$\begin{aligned} l \to 0 : -\ln(l) \to +\infty, N(d_2) \to 1, \frac{1}{l} N(-d_1) \to 0 \\ \Rightarrow s \to 0 \end{aligned}$$

When leverage goes to 1, the debt is equal to the forward value of the assets:

$$\begin{aligned} l \to 1 : d_1 \to \frac{\sigma\sqrt{T}}{2}, d_2 \to -\frac{\sigma\sqrt{T}}{2} \\ \Rightarrow s \to -\frac{1}{T} \ln \left(2N \left(-\frac{\sigma\sqrt{T}}{2} \right) \right) \end{aligned}$$

which is finite (about 400bp for $\sigma = 10\%$ and $T = 1$) but increases with the volatility.

Question (c). With $r = 0$, $F_T = S_0$ and values are expressed as a fraction of S_0:

σ	l	equity	risky debt	spread s
20%	1/2	0.500	0.500	$\approx$ 0bp
20%	3/4	0.256	0.744	78bp
40%	1/2	0.505	0.495	95bp

With low leverage and low volatility, the probability of bankruptcy is negligible: equity is worth $1 - l$ and the debt is riskless. The spread increases both with leverage and with volatility.

Question (d). When σ increases, the value of the call increases, hence the value of equity increases. The value of the put increases as well, hence

the value of the risky debt decreases: shareholders have an interest in the company taking more risk, at the expense of creditors.

Question (e). Before the dividend, the equity is worth $0.273\,S_0$ and the spread is 315bp. The dividend reduces the value of the company to $0.95\,S_0$, hence its forward value to $F'_T = 0.95\,F_T$ and the leverage increases to $l' = \frac{3/4}{0.95} \simeq 0.79$. The equity is then worth $0.231\,S_0$ and the spread increases to 421bp. Shareholders receive $0.05\,S_0 + 0.231\,S_0 = 0.281\,S_0$, more than the $0.273\,S_0$ they had: the dividend transfers wealth from creditors to shareholders. The risky debt is now worth $0.719\,S_0$ instead of $0.727\,S_0$.

2.2 Structured Products

Exercise 17

We want to create a portfolio whose final value would be at least equal to its initial value V_0. To do so, we aim at buying a zero-coupon bond and some call options on the S&P 500. Let r be the riskless interest rate, T the maturity, σ the volatility of the S&P 500 and d its dividend yield.

(a) What notional amount of zero-coupon bond should be bought to ensure principal protection? Let D_0 be the initial value of this zero-coupon bond. Express D_0 as a function of V_0, r and T. Define the "cushion" as $C_0 \equiv V_0 - D_0$. Compute C_0 when $r = 4\%$ and $T = 4$.

(b) We consider buying call options on the S&P 500. Let S_0 be the price of the S&P today and K the selected strike price. We want to obtain the following payoff:

- if $S_T > S_0$: $kV_0 \left(\frac{S_T}{S_0} - 1\right)$;
- if $S_T < S_0$: 0.

Write the payoff. How many calls, with which characteristics, do you need to obtain this payoff? Determine k such that the price of those calls equals C_0.

(c) Find k when $r = 4\%$, $T = 4$, $\sigma = 15\%$ and $d = 1.88\%$. Describe the payoff qualitatively. Compare with buying the S&P directly. Why is this not an arbitrage?

Solution 17

(a) $D_0 = V_0 e^{-rT}$, hence

$$C_0 = V_0 \left(1 - e^{-rT}\right).$$

In the example, with $V_0 = 1$, $C_0 = 14.79\%$.

(b) If one buys n at-the-money calls, the payoff is

$$n\left(S_T - S_0\right)^+$$

so that with $n = \frac{kV_0}{S_0}$ we obtain

$$kV_0\left(\frac{S_T}{S_0} - 1\right)^+$$

which is the required payoff. k is then obtained by the budget equality:

$$C_0 = kV_0 Call\left(S_0 = 1, K = 1, \sigma, T\right)$$
$$k = \frac{1 - e^{-rT}}{Call\left(S_0 = 1, K = 1, \sigma, T\right)}$$

(c) The price of the call equals 14.8%, hence $k = 100\%$. This product provides the performance of the S&P 500 when it is positive, and zero (i.e. the capital back) when it is negative. It looks like an arbitrage but it is not: the structured product does not receive the dividends, whereas a direct investment in the S&P 500 would.

2.3 Hedging a Corporate FX Exposure

Exercise 18

Background: a multinational corporation (MNC) based in France expects to receive USD 100 million from a U.S. client in 3 months. The CFO is concerned about exchange rate fluctuations, especially after recent unexpected macroeconomic news which led to increased volatility in the USD/EUR market. By USD/EUR, we mean the value of 1 USD in EUR, denoted X, with $X_0 = 0.9$ today. The future (random) value of the USD/EUR rate in 3 months is denoted $\tilde{X}_3$.

According to the CEO, the USD should appreciate against the EUR over the next 3 months, due to expectations of a tighter monetary policy by the FED. On the contrary, the CFO thinks there will be less demand for USD as a safe-haven currency, and hence that the USD should depreciate. They are not very sure about what to do...

US rates are equal to $r_{USD} = 5\%$ and the euro rate is $r_{EUR} = 1\%$. The annual volatility of the USD/EUR is $\sigma = 10\%$. Rates are continuously compounded.

(a) Does X increase or decrease when the USD appreciates? If the CEO is right, should you hedge the risk or not? Same question if the CFO is right.

(b) What is the forward value of the USD/EUR rate? Write the formula and calculate a numerical approximation. In EUR, what difference would it make to change USD 100 million at the spot rate or at the forward rate? If the future value of the USD/EUR equals the forward rate, will it mean that the USD has appreciated or depreciated?

(c) Explain why the volatility of $\tilde{X}_3$ should be equal to 5%. What is the typical fluctuation of the EUR equivalent of USD 100 million in three months?

(d) If you do not want to take any risk, what should you do? Will you obtain the support of the CEO or of the CFO?

(e) MNC hesitates between buying a put option or a call option on the USD/EUR. Which one would be appropriate? Explain why.

(f) Eventually, the company purchases a European put option on USD 50 million with a strike price of 0.9 USD/EUR, expiring in 3 months. The price of that option equals 1.5% of the notional amount. Graph the following values as a function of X_3:

- the EUR value of the USD 100 million in three months;
- the EUR value of the put portfolio, including its premium;
- the sum of both.

(g) Compare this strategy with a full hedge (changing the USD forward). Comment. What would be your own view on the matter?

Solution 18

(a) If the USD appreciates, one gets more euros for 1 USD, hence X increases. If the CEO is right, X should increase and MNC should not hedge. If the CFO is right, the opposite is true.

(b) Since USD rates are higher than euro rates, the forward rate is lower than the spot rate:

$$\begin{aligned} F_{3m} &= X_0 e^{(r_{EUR}-r_{USD})T} \\ &= 0.9 \times e^{-4\%/4} \\ &\approx 0.9 \times (1 - 1\%) \\ &\approx 0.891 \end{aligned}$$

In EUR, this means about EUR 89.1 million instead of EUR 90 million, i.e. a difference of approximately EUR 0.9 million. If the future value of the spot equals the forward, the USD will have depreciated a bit.

(c) The variance of a process with i.i.d. increments is of the form $\sigma^2 T$, hence the standard deviation (volatility) scales as $\sigma\sqrt{T}$. The quarterly standard deviation is thus half of the yearly volatility, i.e. 5%. The typical fluctuation in 3 months is therefore ±5% around EUR 89.1 million, i.e. between approximately EUR 84.6 million and EUR 93.6 million.

(d) No risk means a fixed amount of EUR known today: sell the USD 100 million forward immediately. The CFO would approve, but the CEO would think that MNC is losing the opportunity to benefit from a potential appreciation of the USD against the EUR.

(e) The risk of the company is a decrease of X, hence a put option is suitable.

(f) See Figure 2.1.

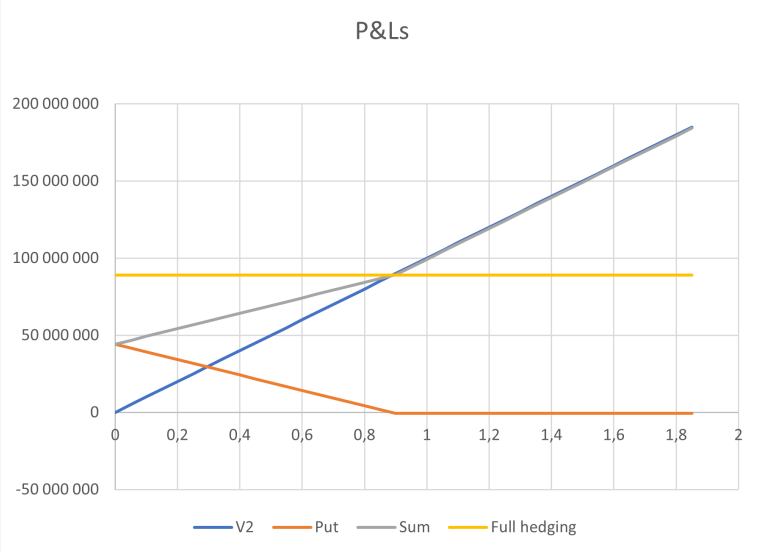


Figure 2.1: Solution of the FX problem

(g) With a full hedge, we would obtain the yellow curve. In my opinion, we do not have specific prediction capabilities, and the risk should be hedged.

2.4 Multi-period / multi-currency financing

Exercise 19

In the following, $r^{eur}(T) = r^{eur}(0,T)$ stands for the continuously compounded euro zero-coupon rate between today and date T. The corresponding discount factor is $P^{eur}(0,T) = e^{-r^{eur}(T)T}$. The yield curve is displayed in Table 2.1. The (unknown) future zero-coupon rate which will prevail at date t for maturity T is denoted $r^{eur}(t,T)$, and the (known) forward rate between dates t and T is denoted $f^{eur}(0,t,T) = f^{eur}(t,T)$.

T	$r^{eur}(T)$
1	3%
2	3.3%
3	3.6%
4	3.7%

Table 2.1: ZC yield curve

A friend of yours needs to borrow 100 000 € for the coming 4 years. He contemplates three possible strategies:

- Strategy SA: contract right now a 4 year ZC loan;
- Strategy SB: borrow money now for the first two years and then, in 2 years time, enter into a new 2 year loan;
- Strategy SC: borrow money now for the first three years and, in 3 years time, enter into a 1 year loan.

He asks you for some advice.

(a) Compute the NPV of each strategy as a function of the different rates.

(b) What is the level of the future ZC rate which makes strategy SB winning or losing as compared to SA? Comment.

(c) Same question for SC.

(d) Eventually, the yield curve remained unchanged during the period. Which strategy has eventually been the most interesting? Discuss.

(e) Propose a Python or an R code which could fill the forward rates matrix of Table 2.2 based on the figures of Table 2.1:

	$T-t=1$	$T-t=2$	$T-t=3$	$T-t=4$
$t=0$				
$t=1$				NA
$t=2$			NA	NA
$t=3$		NA	NA	NA

Table 2.2: Forward rate matrix

Somebody draws your attention to the fact that the Japanese yield curve is flat and equal to 0%. It also seems that the yen keeps on depreciating against the euro (see Figure 2.2). So if the euro value of one yen is lower tomorrow than it is today, borrowing money in this currency seems to make sense (the debt will lose value). Let $Z(t)$ be the exchange rate of 1 JPY in EUR, with $Z(0) = 0.006$ EUR/JPY.



Figure 2.2: EUR/JPY evolution

(f) Describe a mechanism whereby you would borrow in JPY, change your money into EUR, and redeem the loan in JPY by changing the money back from EUR to JPY in 4 years time. What would be the NPV of this scheme as a function of $Z(4)$?

(g) For which value of $Z(4)$ is the NPV of that deal positive? What is the NPV of the gain if $Z(4) = Z(0)$? Comment.

Solution 19

Questions (a) to (c). The three strategies can be parametrised by t, the maturity of the first loan. At t, instead of repaying the loan, one borrows again until date T. If N is the notional amount, the amount to be paid at date t is $\frac{N}{P(0,t)}$ (which is known). At date t, this amount has to be borrowed again until date T, when $\frac{N}{P(0,t)}\frac{1}{P(t,T)}$ will have to be repaid. As a result, the NPV of these strategies is

$$NPV(t) = N\left(1 - \frac{P(0,T)}{P(0,t)\,P(t,T)}\right).$$

Note that $P(t,T)$ is random since $r(t,T)$ is not known today. In terms of interest rates,

$$NPV(t) = N\left(1 - e^{-r(0,T)T + r(0,t)t + r(t,T)(T-t)}\right).$$

The NPV is positive if and only if $-r(0,T)T + r(0,t)t + r(t,T)(T-t) < 0$, i.e.

$$r(t,T) \leq f(t,T) \quad \text{where} \quad f(t,T) := \frac{r(0,T)T - r(0,t)t}{T-t}.$$

The NPV is positive only if the future rate at date t is below the forward rate.

Question (d). If the yield curve does not move, $r(t,T) = r(0,T-t)$. Remark that

$$f(t,T) = r(0,T) + t\frac{r(0,T) - r(0,t)}{T-t}.$$

If the yield curve is upward sloping,

$$r(0,T) \geq r(0,T-t) \quad (\text{because } T > T-t)$$
$$r(0,T) - r(0,t) \geq 0$$

hence $f(t,T) \geq r(0,T-t)$. So, when the yield curve does not move, the realised interest rate is below the forward rate and the strategy is profitable.

Question (e).

	$T-t=1$	$T-t=2$	$T-t=3$	$T-t=4$
$t=0$	3%	3.3%	3.6%	3.7%
$t=1$	3.6%	3.9%	3.93%	NA
$t=2$	4.2%	4.1%	NA	NA
$t=3$	4.0%	NA	NA	NA

Questions (f) and (g). To obtain N euros today, you need $\frac{N}{Z(0)}$ yen, which you can borrow until date T. You will have to repay $\frac{N}{Z(0)}\frac{1}{P^{yen}(0,T)}$ yen, which costs $N\frac{Z(T)}{Z(0)}\frac{1}{P^{yen}(0,T)}$ euros. As a result, the NPV is

$$NPV = N\left(1 - \frac{Z(T)}{Z(0)}\frac{P^{eur}(0,T)}{P^{yen}(0,T)}\right)$$
$$= N\left(1 - \frac{Z(T)}{F(0,T)}\right)$$

where $F(0,T) := Z(0)\frac{P^{yen}(0,T)}{P^{eur}(0,T)} = Z(0)e^{T(r^{eur}(T) - r^{yen}(T))}$ is the forward exchange rate. The NPV is positive if and only if

$$Z(T) < F(0,T),$$

i.e. if the future exchange rate is lower than the forward exchange rate.

In our example, $P^{yen}(0,T) = 1$. If the yen follows the trend indicated by the graph, $Z(4)$ might not increase, and could very well decrease, generating a gain. If the yen depreciates by 30% over the coming 4 years, $Z(4) = 70\%Z(0)$ and the NPV is above 30% of the notional.

2.5 Benefiting from the Abenomics

Exercise 20

In 2012 and 2013, the Japanese government led by Shinzo Abe adopted massive stimulus plans, sometimes referred to as Abenomics.

As a result, the Nikkei 225 appreciated by approx. +57% in 2013. On the contrary, the yen depreciated by approx. -22% against the USD, moving from 87 JPY/USD to 105 JPY/USD. In 2013, interest rates were approximately equal to zero both in JPY and in USD.

How could a US investor benefit from the Abenomics? Consider the two following strategies:

- Strategy 1:
  - Jan 1st, 2013: borrow USD 1M, change it spot into JPY and buy a Nikkei 225 ETF;
  - Dec 31st, 2013: sell the Nikkei ETF, change the proceeds spot into USD and repay the USD loan.
- Strategy 2:
  - Jan 1st, 2013: borrow JPY 87M and buy a Nikkei 225 ETF;
  - Dec 31st, 2013: sell the Nikkei ETF, repay the JPY loan and change the proceeds spot into USD.

(a) Compute the final USD value of both strategies. Discuss.

(b) Consider the two following strategies:

- Strategy 3:
  - Jan 1st, 2013: "buy" a 1Y futures contract on the Nikkei 225 with notional JPY 87M;
  - Dec 31st, 2013: receive the futures payment at expiry and change it spot into USD.
- Strategy 4:
  - Jan 1st, 2013: "buy" a 1Y CME futures contract on USD/JPY with notional USD 1M;
  - Dec 31st, 2013: receive the futures payment at expiry.

Compute the P&Ls of strategies 3 and 4. Can you combine them?

(c) Eventually, what would have been an appropriate Abenomics "trade"?

Solution 20

With zero interest rates, borrowing costs nothing and the futures price of the Nikkei is equal to its spot price. The Nikkei multiplies by 1.57 in JPY and the USD/JPY moves from 87 to 105.

(a) Strategy 1: the USD 1M becomes JPY 87M, invested in the Nikkei and worth JPY $87 \times 1.57 = 136.59$M at the end of the year, i.e. USD $136.59/105 = 1.301$M. After repaying the USD loan, the P&L is +USD 0.301M (+30%): the rise of the Nikkei is largely eaten up by the fall of the yen, since the whole position is exposed to the currency.

Strategy 2: the JPY 87M loan is invested in the Nikkei, worth JPY 136.59M at the end of the year. After repaying the JPY loan, JPY 49.59M remain, i.e. USD $49.59/105 = 0.472$M (+47%). Borrowing in yen hedges the principal against the currency risk: only the gain is exposed to the fall of the yen.

(b) Strategy 3: the long futures position on the Nikkei pays $87 \times 0.57 =$ JPY 49.59M at expiry, i.e. USD 0.472M: it is equivalent to strategy 2 (a futures contract is a position financed in the currency of the underlying).

Strategy 4: the long position on USD/JPY pays $1 \times (105 - 87) =$ JPY 18M, i.e. USD $18/105 = 0.171$M (+17%).

The two strategies can be combined, since neither requires an initial investment: the P&L is then USD $0.472 + 0.171 = 0.644$M.

(c) The appropriate "trade" was to **buy the Nikkei and sell the yen**, both through futures: long Nikkei futures and long USD/JPY futures (i.e. short yen). The equity view and the currency view are then implemented separately, without being exposed to the fall of the yen on the equity position: each leg earns its own performance.