UNIVERSITÉ PARIS 1
PANTHÉON SORBONNE

Advanced Topics in Financial Modelling

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1 Summary

This course provides an introduction to portfolio optimization methods with a focus on the links between option hedging and dynamic portfolio management. Students will build upon the classical mean-variance framework, extend it to multi-period and continuous-time settings, and explore equilibrium models, transaction costs, risk budgeting, and the interplay between optimal control methods and martingale approaches in finance.

2 Prerequisites

- Solid understanding of undergraduate-level finance: basic asset pricing, risk-neutral valuation, and standard portfolio theory.
- Familiarity with stochastic calculus: Brownian motions, Itô's lemma.
- Comfort with advanced calculus, linear algebra, and probability theory.
- Prior exposure to optimization techniques and PDEs is beneficial.

3 Learning Objectives

By the end of the course, students should be able to:

1. Formulate and solve static and dynamic portfolio optimization problems (including the Markowitz framework and multi-asset scenarios).
2. Understand and apply two-fund separation theorems and identify minimum-variance portfolios in various covariance structures.
3. Decompose portfolio returns into option-like profiles and interpret trading impacts.
4. Integrate transaction costs into optimization and derive "no-trade" regions.

5. Derive and interpret the Greeks of dynamic hedging strategies and understand gamma costs, theta, and volatility exposure.

6. Assess the robustness of the Black-Scholes framework and understand the implications of volatility/model misspecification.

4 Course Outline

4.1 Multivariate Portfolio Analysis

- The wealth equation in a multi-asset setting
- Optimal portfolios: $\overline{\alpha} = \frac{1}{\lambda}\Sigma^{-1}\pi$
- Two-fund separation theorem
- Risk Contributions and Euler Decomposition
- Eigen-decomposition of covariance matrices (eigen-portfolios)
- Risk budgeting frameworks

4.2 Transaction Costs and Sparse Portfolios

- Incorporating rebalancing penalties into optimization
- Sub-differentials and optimality conditions
- "No-trade" zones and quadratic programming (OSQP)

4.3 Dynamic Hedging of Options

- Futures-based pricing and martingale measures
- Greeks $(\delta, \gamma, \theta, \nu)$ and economic interpretation
- Dynamic replication strategies
- Robustness of Black-Scholes and Model Misspecification

4.4 Analysis of Specific Dynamic Strategies

- Strategies as option profiles: convex vs. concave
- Contrarian strategies, volatility arbitrage, doubling strategies
- Constant proportion portfolios
- Stop-loss overlays vs. put options

4.5 Normative Dynamic Programming (Samuelson-Merton)

- HJB equation and dynamic optimal control
- CARA, CRRA utilities and closed-form solutions
- Comparative statics and multi-period extensions

4.6 Martingale Approach and Optimal Portfolios

- Cox-Huang and Karatzas-Lehoczky-Shreve results
- Relating optimal payoffs to density processes
- PDE/martingale solution equivalences

5 Note to students

Students are encouraged to use Python, R, or Matlab for computational exercises. OSQP may be used for quadratic programming tasks.

This is an advanced course. Active engagement, completion of readings, and consistent practice are essential for taking the full benefit from it.

Assessment will take the form of a research-style IT project on a chosen advanced topic or strategy, written as a Jupiter notebook or R Markdown document.

References

[1] Benjamin Bruder and Nicolas Gaussel. Risk-return analysis of dynamic investment strategies. *Available at SSRN 2465623*, 2011.

[2] Ioannis Karatzas, Steven E Shreve, I Karatzas, and Steven E Shreve. *Methods of mathematical finance*, volume 39. Springer, 1998.

[3] Nicole El Karoui, Monique Jeanblanc-Picquè, and Steven E Shreve. Robustness of the black and scholes formula. *Mathematical finance*, 8(2):93–126, 1998.

[4] Robert C Merton. Theory of rational option pricing. *The Bell Journal of economics and management science*, pages 141–183, 1973.

[5] Robert C. Merton. *Continuous-time finance*. Blackwell Boston, 1998.